Московский Государственный Университет имени М.В.Ломоносова
Факультет Вычислительной Математики и Кибернетики
Домашняя контрольная работа по вычислению производных

Вычислить производную функции $y = y(x)$. Указать область существования производной:

1. $y = \dfrac{5x}{1-x^3}$;

2. $y = \dfrac{x^2-7x+8}{1-3x+x^2}$;

3. $y = \dfrac{2x}{(1-x)^4(1+2x)^2}$;

4. $y = \dfrac{(3-2x^2)(7-x^3)}{(1+x)^5}$;

5. $y = \dfrac{1+x-x^2}{1-x+x^2}$;

6. $y = \dfrac{x(2+x)}{(1-x)^2(1+x)^3}$;

7. $y = \dfrac{(1-x)^a}{(1+x)^b}$;

8. $y = \dfrac{x^a(1-x)^b}{(1+x)^c}$;

9. $y = x + \sqrt[3]{x} + \sqrt[4]{x}$;

10. $y = x^3\sqrt[3]{x^2} + x^7\sqrt[3]{x}$;

11. $y = \sqrt[3]{x^2} - \dfrac{2}{\sqrt{x}}$;

12. $y = x\sqrt{1+x^2}$;

13. $y = \dfrac{x}{a^2-x^2}$;

14. $y = \sqrt[3]{\dfrac{1+x^3}{1-x^3}}$;

15. $y = \sqrt{x+\sqrt{x+\sqrt{x}}}$;

16. $y = (1+x)\cdot\sqrt{2+x^2}\cdot\sqrt[3]{3+x^3}$;

17. $y = \sqrt[m+n]{(1-x)^m\cdot(1+x)^n}$;

18. $y = \dfrac{1}{\sqrt{1+x^2}(x+\sqrt{1+x^2})}$;

19. $y = \sqrt[3]{1+\sqrt[3]{1+\sqrt[3]{x}}}$;

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20. $y = 5x\cos x$;

21. $y = \cos 2x - 2\sin x$;

22. $y = \sin^n x\cdot\cos nx$;

23. $y = \sin\big(\sin(\sin x)\big)$;

24. $y = \dfrac{\sin^2 x}{\sin x^2}$;

25. $y = \dfrac{\cos x}{2\sin^2 x}$;

26. $y = \dfrac{1}{\sin^3 2x}$;

27. $y = \dfrac{\sin x - x\cos x}{\cos x + x\sin x}$;

28. $y = \operatorname{tg}\frac{x}{2} - \operatorname{ctg}\frac{x}{2}$;

29. $y = \sin\big(\cos^2 x\big)\cdot\cos\big(\sin^2 x\big)$;

30. $y = (2-x^2)\cos x + 2x\sin x$;

31. $y = \operatorname{tg} x - \dfrac{\operatorname{tg}^3 x}{3} + \dfrac{tg^5 x}{5}$;

32. $y = 4\sqrt[3]{\operatorname{ctg}^2 x} + \sqrt[3]{\operatorname{ctg}^8 x}$;

33. $y = \dfrac{1}{\cos^2(x/a)} + \dfrac{1}{\sin^2(x/a)}$;

34. $y = \sin\big(\cos^2(\operatorname{tg}^3 x)\big)$;

35. $y = (a\cos x + b\sin x)^\alpha$;

36. $y = Ae^{-k^2x}\sin(\omega x + \alpha)$;

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37. $y = e^{-x^2}$;

38. $y = 2^{\operatorname{tg}\frac{1}{x}}$;

39. $y = e^x(x^2-2x+2)$;

40. $y = e^x\big(1+\operatorname{ctg}\frac{x}{2}\big)$;

41. $y = \dfrac{\ln 3\cdot\sin x + \cos x}{3^x}$;

42. $y = e^x + e^{e^x} + e^{e^{e^x}}$;

43. $y = 3^{\ln^2(1+e^{-x})}$;

44. $y = \ln^3(x^2)$;

45. $y = \ln\big(\ln(\ln x)\big)$;

46. $y = \ln\big(\ln^2(\ln^3 x)\big)$;

47. $y = \ln\operatorname{tg} x + \dfrac{\operatorname{ctg} 2x}{2}$;

48. $y = \log_2\dfrac{\cos x + x\sin x}{\sin x - x\cos x}$;

49. $y = \left(\dfrac{1-x^2}{2}\sin x - \dfrac{(1+x)^2}{2}\cos x\right)e^{-x}$;

50. $y = \left(\dfrac{a}{b}\right)^x \left(\dfrac{b}{x}\right)^a \left(\dfrac{x}{a}\right)^b$, $a>0,\ b>0$;

51. $y = e^{ax}\cdot\dfrac{a\sin bx - b\cos bx}{\sqrt{a^2+b^2}}$;

52. $y = x^{a^a} + a^{x^a} + a^{a^x}$, $a>0$;

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53. $y = \dfrac{1}{4}\cdot\ln\dfrac{x^2-1}{x^2+1}$;

54. $y = \dfrac{1}{2\sqrt{6}}\ln\dfrac{x\sqrt{3}-\sqrt{2}}{x\sqrt{3}+\sqrt{2}}$;

55. $y = \dfrac{\ln\left(x+\sqrt{x^2+1}\right)}{\sqrt{5}}$;

56. $y = \dfrac{\ln(1+x)}{2} - \dfrac{\ln(1+x^2)}{4} - \dfrac{1}{2(1+x)}$;

57. $y = \dfrac{1}{4(x^4+1)} + \dfrac{1}{4}\cdot\ln\dfrac{x^4}{1+x^4}$;

58. $y = \sqrt{x^2+1} - \ln\left(\frac{1}{x} + \sqrt{1+\frac{1}{x^2}}\right)$;

59. $y = \sqrt{x+1} - \ln\left(1+\sqrt{x+1}\right)$;

60. $y = \dfrac{x}{2}\sqrt{x^2+a^2} + \dfrac{a^2}{2}\cdot\ln\left(x+\sqrt{x^2+a^2}\right)$;

61. $y = x\cdot\ln\left(x+\sqrt{1+x^2}\right) - \sqrt{1+x^2}$;

62. $y = \dfrac{2+3x^2}{x^4}\sqrt{1-x^2} + 3\ln\dfrac{1+\sqrt{1-x^2}}{x}$;

63. $y = \dfrac{\ln\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}$;

64. $y = \dfrac{1}{2\sqrt{ab}}\cdot\ln\dfrac{\sqrt{a}+x\sqrt{b}}{\sqrt{a}-x\sqrt{b}}$, a >0, b>0;

65. $y = \dfrac{1}{2\sqrt{6}}\cdot\ln\left(\dfrac{\sqrt{2}+x\sqrt{3}}{\sqrt{2}-x\sqrt{3}}\right)^2$;

66. $y = 2x\cdot\ln\left(2x+\sqrt{4x^2+1}\right) - \sqrt{4x^2+1}$;

67. $y = \dfrac{1}{1-k}\cdot\ln\dfrac{1+x}{1-x} + \dfrac{\sqrt{k}}{1-k}\cdot\ln\dfrac{1+x\sqrt{k}}{1-x\sqrt{k}}$, $0<k<1$;

68. $y = x\cdot\ln^2\left(x+\sqrt{1+x^2}\right) - 2\sqrt{1+x^2}\cdot\ln\left(x+\sqrt{1+x^2}\right) + 2x$;

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69. $y = \ln\operatorname{tg}\frac{x}{2}$;

70. $y = \ln\left(\operatorname{tg}\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)$;

71. $y = \dfrac{1}{2}\operatorname{ctg}^2 x + \ln\left(\sin x\right)$;

72. $y = \ln\sqrt{\dfrac{1-\sin x}{1+\sin x}}$;

73. $y = \dfrac{\ln 1/x}{4x^4} - \dfrac{1}{16x^4}$;

74. $y = \ln\left(\arccos\frac{1}{\sqrt{x}}\right)$;

75. $y = \dfrac{\ln^3 x + 3\ln^2 x + 6\ln x + 6}{x}$;

76. $y = -\dfrac{\cos x}{2\sin^2 x} + \ln\sqrt{\dfrac{1+\cos x}{\sin x}}$;

77. $y = \dfrac{3}{2}\left(1-\sqrt[3]{1+x^2}\right)^2 + 3\ln\left(1+\sqrt[3]{1+x^2}\right)$;

78. $y = \ln\left(\frac{1}{x} + \ln\left(\frac{1}{x} + \ln\frac{1}{x}\right)\right)$;

79. $y = x\cdot\left(\sin(\ln x) - \cos(\ln x)\right)$;

80. $y = \ln\left(\operatorname{tg}\frac{x}{2}\right) - \cos x\cdot\ln\left(\operatorname{tg} x\right)$;

81. $y = \ln\left(1+\sin^2 x\right) - 2\sin x\cdot\operatorname{arctg}(\sin x)$;

82. $y = \ln\dfrac{x+a}{\sqrt{x^2+b^2}} + \dfrac{a}{b}\cdot\operatorname{arctg}\dfrac{x}{b}$;

83. $y = \ln\dfrac{b+a\cos x+\sqrt{b^2-a^2}\sin x}{a+b\cos x}$, $0\leqslant|a|<|b|$;

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84. $y = \arcsin\dfrac{x}{2}$;

85. $y = \arccos\dfrac{1-x}{\sqrt{2}}$;

86. $y = \operatorname{arctg}\dfrac{x^2}{a}$;

87. $y = \dfrac{1}{\sqrt{2}} \cdot \operatorname{arcctg} \dfrac{\sqrt{2}}{x}$;

88. $y = \sqrt{x} - \operatorname{arctg} \sqrt{x}$;

89. $y = \dfrac{1 + x^2 \cdot \operatorname{arctg} x^2}{\sqrt{1 + x^4}}$;

90. $y = \arcsin(\sin x)$;

91. $y = \arccos(\cos^2 x)$;

92. $y = \arcsin(\sin x - \cos x)$;

93. $y = \arccos \sqrt{1 - x^2}$;

94. $y = \operatorname{arctg} \dfrac{1 + x}{1 - x}$;

95. $y = \operatorname{arcctg}\left(\dfrac{\sin x + \cos x}{\sin x - \cos x}\right)$;

96. $y = \arcsin \dfrac{1 - x^2}{1 + x^2}$;

97. $y = \dfrac{1}{\arccos^2(x^2)}$;

98. $y = \operatorname{arctg} x + \dfrac{\operatorname{arctg} x^3}{3}$;

99. $y = \dfrac{1}{\sqrt{2}} \cdot \ln\left(\dfrac{x - \sqrt{2}}{x + \sqrt{2}}\right)^2 - \dfrac{4}{\sqrt{3}} \cdot \operatorname{arctg} \dfrac{x}{\sqrt{3}}$;

100. $y = x \cdot \arcsin \sqrt{\dfrac{x}{1 + x}} + \operatorname{arctg} \sqrt{x} - \sqrt{x}$;

101. $y = \dfrac{2}{\sqrt{a^2 - b^2}} \cdot \operatorname{arcctg}\left(\sqrt{\dfrac{a - b}{a + b}} \cdot \operatorname{tg} \dfrac{x}{2}\right), \quad a > b \geqslant 0$;

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102. $y = \dfrac{x}{2}\sqrt{a^2 - x^2} + \dfrac{a^2}{2} \arcsin \dfrac{x}{b}, \quad a > 0$;

103. $y = \dfrac{1}{6} \ln \dfrac{(x+1)^2}{x^2 - x + 1} + \dfrac{1}{\sqrt{3}} \operatorname{arctg} \dfrac{2x - 1}{\sqrt{3}}$;

104. $y = \dfrac{\arccos x}{x} + \dfrac{1}{2} \ln \dfrac{1 - \sqrt{1 - x^2}}{1 + \sqrt{1 - x^2}}$;

105. $y = \operatorname{arctg} \sqrt{x^2 - 1} - \dfrac{\ln x}{\sqrt{x^2 - 1}}$;

106. $y = \dfrac{\arcsin x}{\sqrt{1 - x^2}} + \dfrac{1}{2} \ln \dfrac{1 - x}{1 + x}$;

107. $y = \dfrac{x^6}{1 + x^{12}} - \operatorname{arctg} x^6$;

108. $y = \operatorname{arctg} \dfrac{x}{1 + \sqrt{1 - x^2}}$;

109. $y = \operatorname{arcctg} \dfrac{a - 2x}{2\sqrt{ax - x^2}}, \quad a > 0$;

110. $y = \dfrac{3 - x}{2}\sqrt{1 - 2x - x^2} + 2 \arcsin \dfrac{1 + x}{\sqrt{2}}$;

111. $y = \dfrac{1}{4} \ln \dfrac{\sqrt[4]{1 + x^4} + x}{\sqrt[4]{1 + x^4} - x} - \dfrac{1}{2} \operatorname{arctg} \dfrac{\sqrt[4]{1 + x^4}}{x}$;

112. $y = \operatorname{arctg}(\operatorname{tg}^2 x)$;

113. $y = x(\arcsin x)^2 + 2\sqrt{1 - x^2} \arcsin x - 2x$;

114. $y = \dfrac{1}{12} \ln \dfrac{x^4 - x^2 + 1}{(x^2 + 1)^2} - \dfrac{1}{2\sqrt{3}} \operatorname{arctg} \dfrac{\sqrt{3}}{2x^2 - 1}$;

115. $y = \arccos \dfrac{x^{2n} - 1}{x^{2n} + 1}, \quad n \in \mathbb{N}$;

116. $y = \ln \dfrac{1 - \sqrt[3]{x}}{\sqrt{1 + \sqrt[3]{x} + \sqrt[3]{x^2}}} + \sqrt{3} \cdot \operatorname{arctg} \dfrac{1 + 2\sqrt[3]{x}}{\sqrt{3}}$;

117. $y = \dfrac{1}{4\sqrt{2}} \cdot \ln \dfrac{x^2 + x\sqrt{2} + 1}{x^2 - x\sqrt{2} + 1} - \dfrac{1}{2\sqrt{2}} \cdot \operatorname{arctg} \dfrac{x\sqrt{2}}{x^2 - 1}$;

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118. $y = x \operatorname{arctg} x - \ln(1 + x)^2 - \dfrac{1}{2}(\operatorname{arctg} x)^2$;

119. $y = \ln(e^x + \sqrt{1 + e^{2x}})$;

120. $y = \operatorname{arctg}(x + \sqrt{1 + x^2})$;

121. $y = \arcsin\left(\frac{\sin a \sin x}{1 - \cos a \cos x}\right)$;

122. $y = \dfrac{x\sqrt{1 - x^2}}{1 + x^2} - \dfrac{3}{\sqrt{2}} \operatorname{arcctg} \dfrac{x\sqrt{2}}{\sqrt{1 - x^2}}$;

123. $y = \operatorname{arctg} e^x - \ln \sqrt{\dfrac{e^{2x}}{e^{2x} + 1}}$;

124. $y = \arccos\left(\sin x^2 - \cos x^2\right)$;

125. $y = \arcsin\left(\sin x^2\right) + \arccos\left(\cos x^2\right)$;

126. $y = \sqrt{1 + \sqrt[3]{1 + \sqrt[4]{1 + x^4}}}$;

127. $y = \ln^2\left(\sec\left(2^{\sqrt[3]{x}}\right)\right)$;

128. $y = \operatorname{arcctg} \dfrac{1}{\sqrt{\operatorname{ctg} \frac{1}{x^2}}}$;

129. $y = \left(\operatorname{ctg} \dfrac{x}{3}\right)^{\sqrt[3]{e^x - e^{-x}}}$;

130. $y = \dfrac{1}{4\sqrt{3}} \cdot \ln \dfrac{\sqrt{x^2+2} - x\sqrt{3}}{\sqrt{x^2+2} + x\sqrt{3}} + \dfrac{1}{2} \operatorname{arctg} \dfrac{\sqrt{x^2+2}}{x}$;

131. $y = \dfrac{1}{2\sqrt{2}} \operatorname{arctg} \dfrac{x\sqrt{2}}{\sqrt{1+x^4}} - \dfrac{1}{4\sqrt{2}} \cdot \ln \dfrac{\sqrt{1+x^4} - x\sqrt{2}}{\sqrt{1+x^4} + x\sqrt{2}}$;

132. $y = e^{m \arcsin x} \cdot \big(\cos(m \arcsin x) + \sin(m \arcsin x)\big)$;

133. $y = \sqrt{1-x^2} \cdot \ln \sqrt{\dfrac{1-x}{1+x}} + \dfrac{1}{2} \ln \dfrac{1-\sqrt{1-x^2}}{1+\sqrt{1-x^2}} + \sqrt{1-x^2} + \arcsin x$;

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134. $y = \sqrt[x]{x}, \quad x > 0$;

135. $y = (\operatorname{arctg} 4x)^{\sqrt[3]{1+x^2}}$;

136. $y = (\ln x)^x : x^{\ln x}$;

137. $y = \operatorname{arctg}(\operatorname{th} x)$;

138. $y = \log_x e$;

139. $y = (\arcsin x)^{\frac{\sin x}{x}}$;

140. $y = \dfrac{\operatorname{ch} x}{\operatorname{sh}^2 x} - \dfrac{1}{2\operatorname{ch}^2 x}$;

141. $y = \dfrac{\operatorname{ch} x}{\operatorname{sh}^2 x} - \ln\big(\operatorname{cth} \frac{x}{2}\big)$;

142. $y = \left(\dfrac{\arcsin\big(\sin^2 x\big)}{\arccos\big(\cos^2 x\big)}\right)$;

143. $y = \arccos\left(\dfrac{1}{\operatorname{ch} x}\right)$;

144. $y = \ln(\operatorname{ch} x) + \dfrac{1}{2\operatorname{ch}^2 x}$;

145. $y = \left(\operatorname{tg} \dfrac{x}{2}\right)^{x \arcsin 2x}$;

146. $y = x + x^x + x^{x^x}, \quad x > 0$;

147. $y = x^{x^a} + x^{a^x} + a^{x^x}, \quad a > 0,\ x > 0$;

148. $y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$;

149. $y = \big(\arcsin(\sin^2 x)\big)^{\operatorname{arctg} x}$;

150. $y = \dfrac{b}{a}x + \dfrac{2\sqrt{a^2-b^2}}{a} \cdot \operatorname{arctg}\left(\sqrt{\dfrac{a-b}{a+b}} \cdot \operatorname{th} \dfrac{x}{2}\right), \quad 0 \leqslant |b| < a$;

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151. Найдите производную функции $y = \ln\big(\cos^2 x + \sqrt{1+\cos^4 x}\big)$, вводя промежуточное переменное $u = \cos^2 x$.

Приемом, указанным в примере *(138)*, найдите производные функций:

152. $y = \dfrac{e^{-x^2} \cdot \arcsin(e^{-x^2})}{\sqrt{1-e^{-2x^2}}} + \dfrac{1}{2}\ln\big(1-e^{-2x^2}\big)$;

153. $y = \dfrac{a^x}{1+a^{2x}} - \dfrac{1-a^{2x}}{1+a^{2x}} \cdot \operatorname{arcctg} a^{-x}$;

154. $y = (\arccos x)^2 \cdot \left(\ln^2(\arccos x) - \ln(\arccos x) + \dfrac{1}{2}\right)$;

155. $y = \dfrac{1}{2}\operatorname{arctg}\big(\sqrt[4]{1+x^4}\big) + \dfrac{1}{4}\ln \dfrac{\sqrt[4]{1+x^4}+1}{\sqrt[4]{1+x^4}-1}$;

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Все ответы максимально упростить.